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Modeling Polycondensation of Lactic Acid

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Summary: A mathematical model for the polycondensation of lactic acid accounting for water removal by diffusion is developed. The corresponding kinetic parameters are estimated by performing experiments under different sets of reaction conditions in open system. The model results are compared with the experimental ones and, after validating the model, parametric simulations are further carried out to understand the effects of operating conditions such as reaction temperature, catalyst activity, and reaction pressure.

Keywords: lactic acid; modeling; polycondensation

Introduction

High molecular weight poly(lactic acid) (PLA), one of the emerging biodegradable polymers, is commercially produced by the ring opening polymerization (ROP) of lactide. However, due to the complexity involved in the process, PLA is not yet able to serve as a commodity plastic.[1] On the other hand, the simpler direct polycondensation of lactic acid usually produces low molecular weight products due to slow reaction kinetics and difficulties in the removal of the by-product (water), which introduces reversibility in the reaction. The two aforementioned ways of manufacturing PLA are operated under different operating conditions, and thus result in various reaction types such as, bulk mode,[2–5] bulk-melt/solid,[6–8] and solution polymerization.[9–10]

The raw material for the ROP, lactide, is produced by controlled oligomerization of lactic acid. This method leads quite easily to high molecular weight products (Mw $\sim$ 100,000–300,000 Da). Many researchers have studied the process experimentally both in the bulk[4,5] and solution mode.[10]

Conversely, the polycondensation of lactic acid usually leads to products with smaller molecular weights (Mw $\sim$ 5,000–30,000 Da). In the literature different ways are suggested to increase the molecular weight: use of chain-extending agent,[2] use of solvent to carry out azeotropic dehydration,[9] polycondensation under melt/solid conditions,[6–8] are few to list. More recently Chen *et al.* (2006)[3] reported that by decompressing the reactor the molecular weight of the produced polymer can be increased. They reported that, by controlling the decompression rate the molecular weight of polymer can be affected significantly.

In the literature though many attempts are reported in understanding and investigating the performance of PLA production through experimental means, there are only few efforts provided in studying the system through mathematical modeling[4,11] and only for the ROP of lactide. Hence, the aim of the present work was to develop a mathematical model for the polycondensation of lactic acid and, most importantly, to evaluate the corresponding model parameters. The model was validated by comparison with experimental results. The model was then used to simulate the performance of the system under different sets of operating conditions.

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Experimental Part

L-lactic acid (90%) was purchased from Fluka Chemicals. The catalyst $SnCl_2$ (98% pure) was purchased from Sigma-Aldrich. Reactions were carried out in a glass reactor (100 ml) with $SnCl_2$ as the catalyst. The reacting mixture was mechanically stirred using a stirrer connected to the motor through a magnetic coupling, thus ensuring no leakage. For reactions in open conditions, one neck of the reactor was connected to flow nitrogen while the other neck was connected to an open condenser to collect the water vapors from the reactor. More details of the experimental setup and reaction procedure can be found in Harshe *et al.* (2007).[12]

The amount of water present in the reaction mixture was measured by KF titration of samples. Namely, 633 Karl Fischer-Automat and 645 Multi-Dosimat have been used, capable of measuring up to 100 ppm of water concentration. About 0.5 g of polymer was used in 10 ml of CH_2Cl_2; Merck Combititrant 5 and Merck Combimethanol were used as titrant and solvent, respectively. Limited solubility of the polymer samples in the titration medium was found at large reaction times, thus affecting the reliability of the measurements.

Model Development and Validation

A mathematical model was developed with the assumption of equal reactivities of the functional groups (–OH and –COOH) present in any molecule in the reacting mixture. The reaction mechanism used in the model development was:

$$A(R_i)B + A(R_j)B \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} A(R_{i+j})B + W$$

where A and B represent the functional groups present in any lactic acid and/or PLA molecule and R is the repeating unit. The subscripts i and j indicate the chain length, while k_1 and k_{-1} stand for the polymerization and depolymerization rate constant, respectively. The water concentration is denoted by W.

With the reaction mechanism defined above, the rate of accumulation of polymer of chain length i can be derived by following the approach detailed by Dotson *et al.* (1996).[13] Assuming constant reaction volume, the time evolution of the concentration of polymer of chain length i can be written as:

$$\frac{dP_i}{dt} = k_1 \sum_{j=1}^{i-1} P_{i-j}P_j + 2k_{-1}W \sum_{j=i+1}^{\infty} P_j - 2k_1 P_i \sum_{j=1}^{\infty} P_j - k_{-1}W(i-1)P \tag{1}$$

$$\text{for } 1 \le i \le \infty$$

The dynamic balance for the water concentration accounting for polymerization, depolymerization, and vaporization can be written as:

$$\frac{dW}{dt} = k_1 \sum_{i=1}^{\infty} P_i \sum_{j=1}^{\infty} P_j - k_{-1}W \sum_{i=1}^{\infty} (i-1)P_i - R_w \tag{2}$$

where R_w is the rate of water removal by vaporization. It is evaluated as:

$$R_w = K_{y,w}\left(\frac{P_w^* x_w}{P_t} - y_w\right) \cong K_{y,w}\left(\frac{P_w^* x_w}{P_t}\right) = K_{x,w}x_w \tag{3}$$

with $K_{y,w}$ and $K_{x,w}$ overall mass transport coefficients, P_w^* vapor pressure of water, P_t total pressure and x_w, y_w mole fractions of water in liquid and gas phase, respectively. Note that, due to specific experimental arrangement (reactions carried out under high flow of inert gas), the mole fraction of water in the gas phase becomes negligible and R_w can be evaluated using the irreversible approximation of the previous equation.

The values of three model parameters are required: the polymerization rate constant (k_1), the depolymerization rate constant (k_{-1}), and the overall mass transfer coefficient for water ($K_{y,w}$).

The values of equilibrium constant, $K_{eq} = k_1/k_{-1}$ reported in ref.[12] as measured through reactions carried out in closed conditions (no water removal) have been used without adjustment. Therefore, two model parameters need to be estimated to complete the kinetic description, a rate coefficient, e.g. k_1, and the transport coefficient, $K_{y,w}$. To this aim, reactions with $SnCl_2$ as the catalyst at four different temperatures (110, 130, 150 and 170 °C) in open conditions (large flow of nitrogen) were carried out. For each reaction, samples were taken and analyzed by using gel permeation chromatography (GPC) to estimate the molecular weight distribution of the polymer, while KF titration was used to calculate the water concentration in the reaction mixture. Such data were simulated by the model and the corresponding parameter values were estimated by best fit to the experimental data. It was experimentally found by analyzing the polymer samples that, the water fraction in the reaction mixture drops below 1% after the first few hours of reaction (5 to 7 hr) at all temperatures. So, the polymerization rate constant k_1 was estimated using the classical expression of the number average molecular weight corresponding to complete water removal (cf. ref.[14]). This way, the following relationship describing the dependence of the polymerization rate constant on the reaction temperature was obtained:

$$\ln(k_1) = -\left(\frac{6810.3}{T}\right) + 6.1928 \tag{4}$$

The remaining parameter, $K_{y,w}$, was estimated by fitting the model predictions to the experimental data. Namely, a single best-fit value was considered, thus neglecting the temperature dependence of the transport phenomenon.

Figure 1 shows the comparison of the experimental and model predicted weight average molecular weight for different reaction temperatures and reaction times. The model predictions fit fairly well the experimental data provided that large enough value of $K_{y,w}$ is used, larger than about 5×10^{-4} [mol/l/s]. This means that the experiments were indeed operated at conditions of fast rate of water removal. Thus, such transport is not limiting and it can be concluded that the reaction kinetics is intrinsically slow and that very large reaction times are required to produce high molecular weight polymer.

The actual value of $K_{y,w}$ (2×10^{-3} [mol/l/s]) was estimated by fitting the experimental values of residual water concentration as

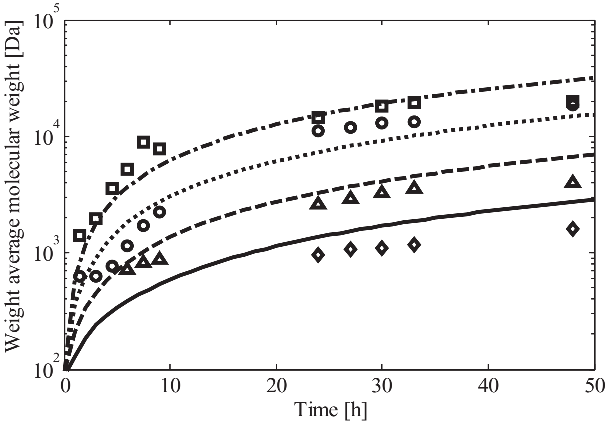

Figure 1.

Comparison of model and experimental results for reactions carried with $SnCl_2$ at different reaction temperatures (points: experiments, lines: model: ◇, —: 110 °C; △, — —: 130 °C; ○, ---: 150 °C; □, -·- : 170 °C) for $K_{y,w} = 5 \times 10^{-4}$ [mol/l/s].

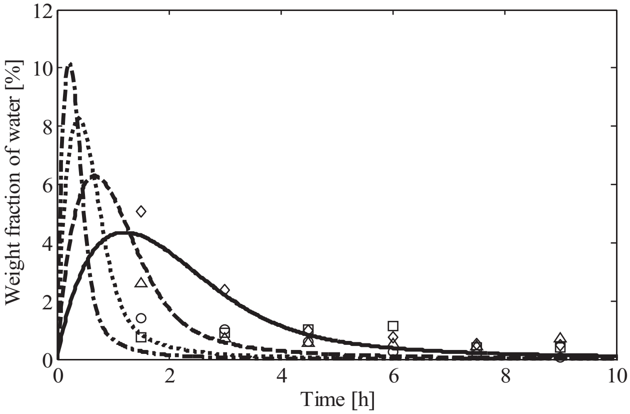

Figure 2.
Water content profile as measured by Karl Fischer and predicted by model for different reaction temperatures (points: experiments, lines: model: ◇, —: 110 °C; △, — —: 130 °C; ○, ---: 150 °C; □, - —: 170 °C) with $K_{y,w} = 2 \times 10^{-3}$ [mol/l/s] and $W_0 = 0$ [mol/l].

measured through KF titration. The comparison is shown in Figure 2 for the first 10 hours of reaction: in fact, the experimental values become more and more inaccurate the longer the reaction time, because the polymer becomes increasingly insoluble into the titration medium.

Even though the agreement is definitely not very accurate, it should be mentioned that an oversimplified description of the diffusion rate is carried out based on the use of a constant, time-independent mass transport coefficient. Actually, due to the continuous increase of the mixture viscosity with time (reflecting the corresponding increase of the polymer molecular weight), a corresponding decrease of transport efficiency is expected, which should result in some shape distortion of the curves of residual water amount shown in the figure. Another possible source of errors could be the unknown initial amount of water: it was assumed negligible in all cases, even though it is quite probable that different residual amounts are present in different experiments. The role of such inaccuracy was checked by parametric simulations and found fully negligible with respect to the calculated molecular weight of the polymer, but affecting the calculated profiles in Figure 2 at an extent which is roughly proportional to the selected initial value.

Parametric Analysis

The model was used to study the effect of different operating conditions, such as water removal rate, catalyst activity, and pressure on the performance of the process.

To further elucidate the effect of water removal rate, the value of the overall mass transfer coefficient was changed, varying the rate of water removal from slow to very fast. As can be seen from Figure 3, the dynamic profile as well as the performance of the process in terms of the weight average molecular weight of the produced polymer at a given temperature and reaction time is a very strong function of the overall mass transfer coefficient, $K_{y,w}$. However, the molecular weight of the polymer becomes practically independent upon the value of the mass transfer coefficient above $K_{y,w} = 5 \times 10^{-4}$ [mol/l/s]. Since a good agreement between the model and experimental results was found for much larger value of the overall mass transfer coefficient, it can be concluded that the process was operating very close to the conditions of instantaneous removal of water. Therefore, for a given catalyst system and reaction temperature, the process performance cannot be improved further, which indicates that a catalyst with

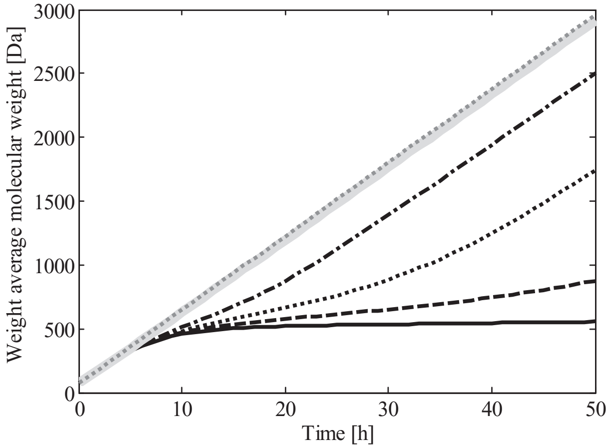

Figure 3.
Effect of overall mass transfer coefficient, $K_{y,w}$ [mol/l/s], on the dynamic profile of the system with $SnCl_2$ catalyst at 110 °C (━━━: 1×0^{-5}; – – –: 5×10^{-5}; - - - - : 1×10^{-4}; – · – : 2×0^{-4}; ▬▬: 5×10^{-4};……: infinite).

larger activity is required to increase the molecular weight of the polymer.

In all previous model calculations, it was assumed the value of the mass transport coefficient, $K_{y,w}$, is constant. However, it is quite likely to have time-variable transport resistances due to the large change of viscosity experienced by the polymer matrix during reaction. The viscosity of a polymer melt is a function of the weight average molecular weight (M_w) of the polymer. Generally, the dependence of the viscosity on the molecular weight can be expressed as[15]:

$$\eta \propto M_w^n \tag{5}$$

In turn, the diffusion coefficient (D) of a species at low molecular weight in such a matrix is in inverse relation to the polymer solution viscosity (η), hence$D \propto 1/\eta$. In Equation (5) the exponent n depends on the polymer molecular size, and it is usually around 1 up to a critical value of the molecular weight specific for each polymer, while becoming larger for longer chains. Accordingly, the mass transfer coefficient for water removal is no more a constant, and it becomes a function of the polymer molecular weight through the following relationship:

$$K_{y,w} = \frac{K_{y,w}^0 F}{M_w^n} \tag{6}$$

where $K_{y,w}^0 = 5 \times 10^{-4}$ [mol/l/s], which is a function of the liquid-film thickness (δ), and is constant assuming constant δ, while F is a factor. Moreover it should be noted that for $F = 1$ and $n = 0$ the value of $K_{y,w}$ reduces to the constant value of $K_{y,w}^0$.

To investigate the effect of such dependence on the process performance, parametric simulations were carried out evaluating $K_{y,w}$ through Equation (6) and for $n = 1$. The latter condition should be reasonable because the typical range of values of the critical molecular weight is 2,000 to 60,000 Da and the reaction at the minimum temperature (110 °C) was considered in this case. Figure 4 shows the water concentration profile for different values of F: while the initial part of the concentration profile is only slightly changed, the decreasing tail is significantly retarded, due to the large resistance experienced by water at increasing polymer molecular weight. Even though the actual value of F could be estimated by fitting the experimental values in Figure 2, the accuracy of such experimental data is not

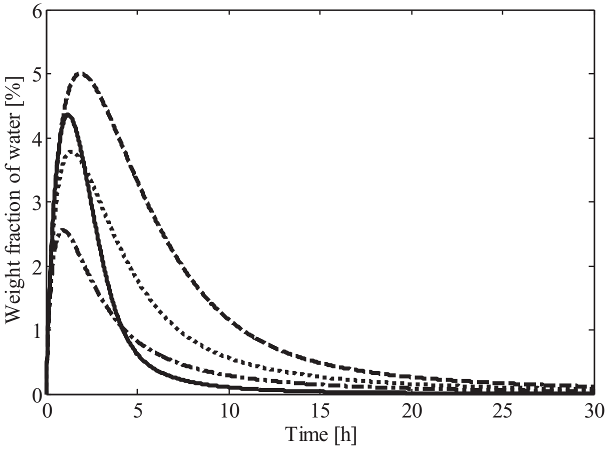

Figure 4.
Water concentration profile for different values of F, (—: $n = 0$ and $F = 1$; and with $n = 1$; F — —: 0.2; • • • • • •: 0.3; –•–•– : 0.5).

enough to get reliable evaluations. On the other hand, this same dependence does not have any significant effect on the predicted weight average molecular weight (Figure 5) and the value of M_w is almost independent upon the specific F value. Therefore, such dependence has been neglected in all the rest of this work.

Now, we investigate the combined effect of catalyst activity and mass transfer coefficient on the polymer molecular weight, which is shown in Figure 6. As expected, the molecular weight of the polymer increases with increasing the reaction temperature, provided that the rate of water removal is large enough. However, such effect is practically suppressed by increasing the diffusion limitations: when the value of $K_{y,w}$ is so small to have the water diffusion as rate determining step, all curves come together and very similar values of molecular weight are produced whatever the operating temperature.

Under the conditions of complete water removal it was found that the molecular weight of the polymer is function of the

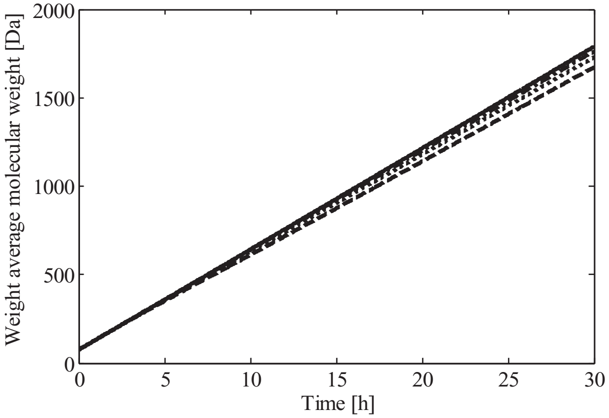

Figure 5.
Dynamic profile of molecular weight for different values of F (—: $n = 0$ and $F = 1$; and with $n = 1$; F — — : 0.2; • • • • • •: 0.3; –•–•– : 0.5).

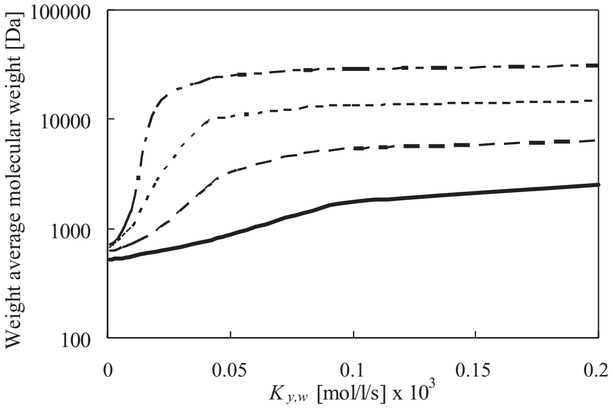

Figure 6.
Effect of overall mass transfer coefficient, $K_{y,w}$, molecular weight of the polymer at different temperatures with $SnCl_2$ as the catalyst after 50 h of reaction time (—:110 °C; – – -:130 °C;- - - - : 150 °C; — · —: 170 °C).

reaction temperature and/or catalyst activity. To analyze the latter effect, parametric simulations were carried out by increasing the pre-exponential factor of the rate coefficient given by Equation (3). The corresponding time evolutions of the weight average molecular weight for the reaction at 170 °C are shown in Figure 7: since water removal is anyhow very fast and the role of depropagation is limited, the final molecular weight scales linearly with the catalyst activity. It can be concluded from the figure that to achieve high molecular weights ($\sim$100,000 Da) in 24 h of reaction for the specific reactor configuration under examination, an increase of catalyst activity of about one order of magnitude is required.

Recently Chen *et al.* (2006)[3] reported that the molecular weight of the polymer can be affected significantly by decompressing the reactor. They reported the highest molecular weight (Mw $\sim$ 130,000 Da) of PLA which was never obtained by direct polycondensation of lactic acid. To this aim they decompressed the reactor very slowly with a decompression rate as low as

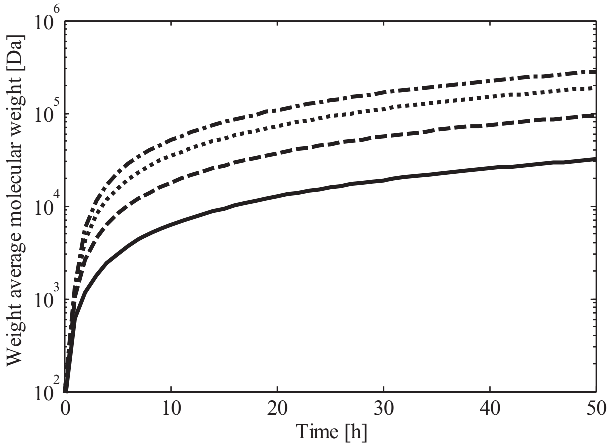

Figure 7.
Effect of catalyst activity on the performance of the process [Catalyst: $SnCl_2$; Temperature: 170 °C] (—: k_1;– – -:$3k_1$;- - - - : $6k_1$; — · —: $9k_1$).

100 mbar/h and reached final pressure of 1 mbar.

We performed simulations to study the effect of direct decompression (instantaneous reduction of pressure) on the molecular weight of the polymer. In agreement with the previous discussion, we found that the system performance is not affected even after decompressing the reactor to a very low final pressure (1 mbar). Such result confirms once more that our system was working very close to the case of instantaneous removal of water. When the same decompression profiles as described by Chen *et al.* (2006)[3] were implemented, no dependence of the molecular weight on the final pressure and/or decompression rate was found. Therefore, since their experimental system cannot remove water at a rate faster than the maximum possible, such a remarkable result cannot be explained.

Conclusion

A dynamic model for the polycondensation of lactic acid accounting for water removal by diffusion was developed. The model and the corresponding parameter values were validated by comparing the model simulation results with the experimental results. The developed model was found to predict with reasonable accuracy the time evolution of average polymer properties. The model was also used to study the effects of different operating conditions on the performance of the system. It was found that the reaction proceeds very close to the case of instantaneous removal of water, and hence there is no effect of decompression and/or decompression profile on the molecular weight of the produced polymer. For the specific reactor under examination, large catalyst activities are essential to achieve high molecular weights.

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